$$
\begin{aligned}
&\langle \beta_1, \ldots \beta_N|^A \sum_i \mathbf{d}_i |\alpha_1, \ldots \alpha_N\rangle^A \\
&\quad = \frac{1}{N!} \sum_i \sum_{PP'} (-1)^{\{P\}+\{P'\}} \langle \beta_{P_1}|\alpha_{P'_1}\rangle \ldots \langle \beta_{P_i}|\mathbf{d}_i|\alpha_{P'_i}\rangle \ldots \langle \beta_{P_N}|\alpha_{P'_N}\rangle \\
&\quad \simeq \frac{1}{N!} \sum_i \sum_{PP'} (-1)^{\{P\}+\{P'\}} \delta_{\beta_{P_1},\alpha_{P'_1}} \ldots \delta_{\beta_{P_{i-1}},\alpha_{P'_{i-1}}} \langle \beta_{P_i}|\mathbf{d}_i|\alpha_{P'_i}\rangle \ldots \delta_{\beta_{P_N},\alpha_{P'_N}} \\
&\quad = \frac{1}{N!} \sum_{iP} \left[\langle \beta_{P_i}|\mathbf{d}_i|\alpha_{P_i}\rangle \prod_{j\neq i} \delta_{\beta_{P_j},\alpha_{P_j}} \right] = \sum_i \left[\langle \beta_i|\mathbf{d}_i|\alpha_i\rangle \prod_{j\neq i} \delta_{\beta_j,\alpha_j} \right] .
\end{aligned}
$$